

Multiobjective Pareto Optimization of Electromagnetic Devices Exploiting Hybrid Kriging

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The paper focuses on resolving the storage issue of correlation matrices generated by kriging surrogate models in the context of electromagnetic optimization problems with many design variables and multiple objectives. A hybrid kriging approach that involves a direct algorithm in kriging is able to maintain memory requirements at a nearly constant level while offering high efficiency of searching for a global optimum. The feasibility and efficiency of this proposed methodology is demonstrated using an example of a classic two-variable analytic function and a new proposed benchmark TEAM multi-objective pareto optimization problem.

Index Terms—Kriging surrogate model, correlation matrices, hybrid kriging, direct algorithm, multi-objective pareto optimization.

I. INTRODUCTION

KRIGING, as a type of regression model, is able to predict a response surface of the objective function through exploiting the spatial correlation of data based only on limited information. However, it was found that large-scale tasks – multi-objective and dealing with many design variables – may lead to a ‘combinatorial explosion’ when all the required correlation matrices are established between the sample points and the design vectors. The partitioning scheme [1] of the correlation matrices, splitting them into manageable sizes, can mitigate to some extent the burden of storing this massive amount of data, but sacrifices may need to be made in terms of computing efficiency at each iteration to achieve more available physical memory. Therefore a more efficient method capable of removing this bottleneck is sought.

II. HYBRID KRIGING

The ‘direct optimization’ algorithm [2], motivated by a modification of Lipschitzian optimization, is able to address difficult global optimization problems with bound constraints. It only requires a decision based on available information where to search next during the optimization process.

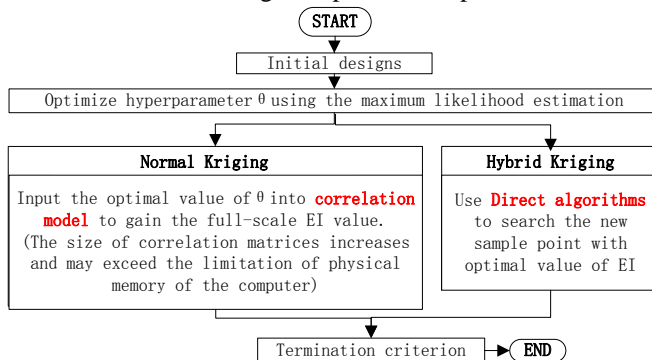


Fig. 1. The decision-making chart for normal kriging and hybrid kriging.

The direct algorithm is utilized to assist kriging in finding the next sampling point with an optimal value of Expected

Improvement [3], rather than building the full-field EI over the whole design space based on very large correlation matrices. This combination of the kriging and the direct algorithm will be referred to as ‘hybrid kriging’. Along with the increase in the number of sampling points selected by kriging throughout the process, the amount of data produced by the hybrid kriging can remain nearly constant. The optimizing procedures of kriging and hybrid kriging are shown in Fig. 1.

III. NUMERICAL EXPERIMENTS

To verify the advantages of the proposed hybrid kriging methodology, a two-variable analytic test function of Fig. 2, with one global minimum and several local minima, has been attempted. The hybrid kriging, requiring only 143 iterations to locate the global minimum (Fig.3), is twice as efficient as the kriging assisted EI that needs 324 iterations [3].

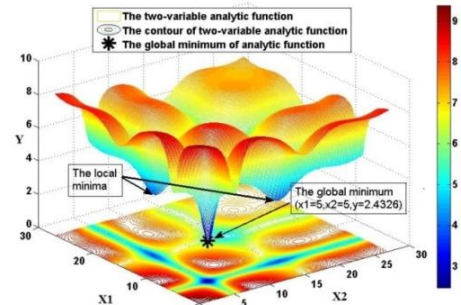


Fig. 1. The two-variable analytic function.

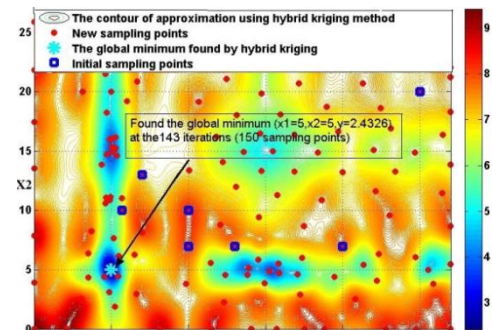


Fig. 2. The highly efficient approximation of hybrid kriging.

More significantly, however, the peak memory occupied by the hybrid kriging at each iteration is maintained at a nearly constant level, whereas the memory consumed by the kriging with EI increases linearly throughout the optimization process. On the other hand, the computing times, simultaneously monitored, show similarity for both the hybrid kriging and the normal kriging.

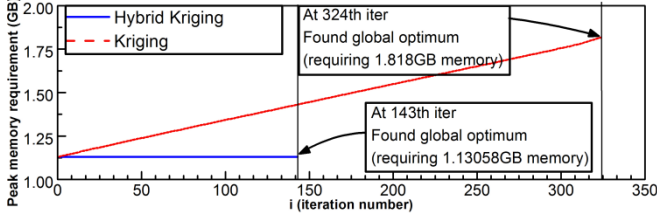


Fig. 3. Monitoring of peak memory requirements of each iteration

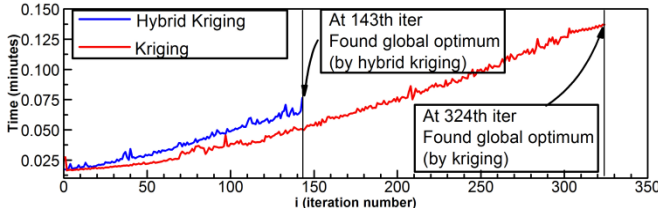


Fig. 4. Monitoring of computing times of each iteration.

IV. BENCHMARK TEAM OPTIMIZATION PROBLEM

The normal kriging, but with different sampling strategies for balancing exploration and exploitation, has been applied to the well-known benchmark TEAM 22 and 25 problems [4]. The results were good but the memory issue clearly visible, making the approach impractical to high-dimensional tasks. The hybrid kriging, with the burden of memory accumulation removed, has been applied to the proposed new multiobjective TEAM benchmark problem with 10 variables, defined as follows. An air-cored single-layer solenoid composed of 20 coils carries a current of density J . The target is to find the optimal distribution of the 20 radii $1\text{mm} < r(z) < 14.5\text{mm}$, $-15\text{mm} < z < 15\text{mm}$, for each turn, that yields the prescribed flux density $B_0(z_q) = 5\text{mT}$ in a sub-region $-5\text{mm} < z < 5\text{mm}$ along the solenoid axis. The flux density at the point z along the solenoid axis may be expressed as

$$B(z) = \frac{\mu_0}{2} \int_{-d}^d \int_{r_i}^{r_s} \frac{J r^2(\xi) dr d\xi}{\sqrt{r^2(\xi) + (z-\xi)^2}} \quad (1)$$

The symmetrically distributed winding is composed of $n_r=20$ series-connected turns, hence ten unknown radii may be identified as design variables. The flux density is prescribed in $n_p=41$ sample points, evenly distributed along the solenoid axis. The two goals are to minimize the discrepancy between the prescribed ($B_0(z_q)$) and the actual field ($B(z_q, r(\xi_l))$) along the solenoid axis, and to minimize the field sensitivity with respect to perturbations in the solenoid radii. A full description of the proposed TEAM problem of an air-cored multi-turn winding will be provided in the extended paper.

The objective functions are defined as

$$f_1(r) = \max_{q=1, n_p} |B(z_q, r(\xi_l)) - B_0(z_q)|, l = 1, n_t \quad (2)$$

$$f_2(r) = \max_{l=1, n_t} [\|B^+ - B(r(\xi_l))\| + \|B(r(\xi_l)) - B^-\|] \quad (3)$$

A popular scalarizing method [5] is applied to assist the hybrid kriging methodology to combine the multiple objectives using a weighted sum (the weights ω_i are set to 1)

$$\text{Minimize } f(x) = \sum_{i=1}^M \omega_i \hat{f}_i(x) \quad (4)$$

Fig. 5 demonstrates the objective function trajectory of sampling points obtained by hybrid kriging. Including one randomly chosen initial point, the hybrid kriging required 201 sampling points. The best results in terms of the minima of f_1 and f_2 are shown in Fig. 5. This test was terminated manually after 200 iterations for better clarity; it is expected that ultimately the termination criterion will be formulated as follows: if the EI of the sampling points declines at a specific value, the hybrid kriging predictor will be stopped.

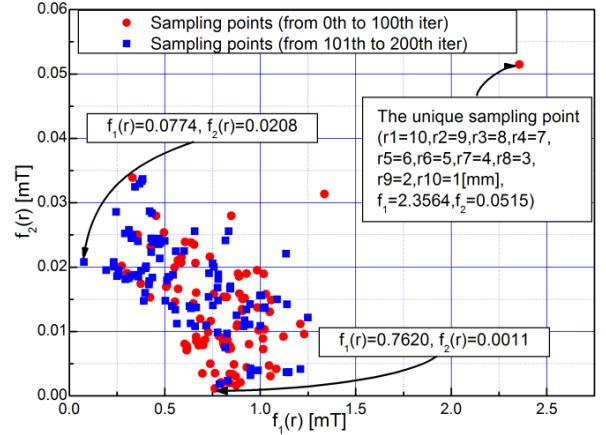


Fig. 5. Objective function trajectory including two goals ($f_1(r)$ and $f_2(r)$).

V. CONCLUSIONS

A novel hybrid kriging model has been proposed to resolve the storage issue of accumulating data during normal kriging. The new algorithm outperforms our previous models in terms of optimization efficiency. A benchmark TEAM problem with 10 design variables for multi-objective Pareto optimization of electromagnetic devices has been utilized to verify the feasibility and efficiency of hybrid kriging. More description of the brute-force sampling of the full-scale objective space and comparison with other multi-objective optimization methods [6] will be provided in the full paper.

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